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SOLUTIONS OF PROBLEMS.

2753 [1919, 73]. Proposed by A. M. HARDING, University of Arkansas.

Through a point P, within a circle, draw a chord which will be trisected at P.

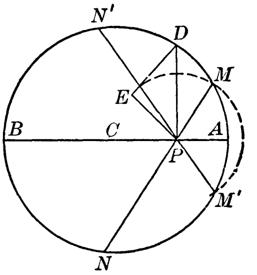
I. Solution by P. J. da Cunha, University of Lisbon.

Soient C le centre et R le rayon du cercle donné, AB le diamètre mené par le point intérieur P, a la distance CP et MN la corde demandée. Comme sur toutes les cordes menées par un point intérieur à un cercle donné, les deux segments compris entre le point et la circonférence ont un produit constant, on a $MP \times PN = AP \times PB$, ou, MP devant être la tiers de MN,

$$2MP^2 = (R - a)(R + a) = R^2 - a^2.$$

Alors, si l'on mène par P une perpendiculaire au diamètre AB, cette perpendiculaire coupe la circonférence à un point D, et l'on a $DP^2 = R^2 - a^2$, et, par conséquent, $2MP^2 = DP^2$.

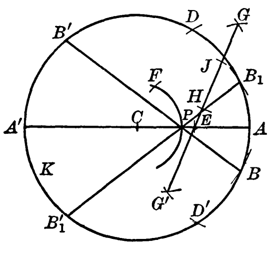
Pour déterminer MP il suffit, donc, de construire un carré dont la diagonale soit DP. Soit PE le coté de ce carré. Du point P comme centre, avec un rayon égale à PE, on décrit un arc qui coupe la circonférence en deux points M et M'. Les cordes MN et $M'N'$ repondent à la question.



II. Solution and Discussion by E. J. Moulton, Northwestern University.

It is obviously not always possible to construct such a chord; for example, there is no such chord if P is at the center of the circle.

Let K be the given circle, C its center and AA' the diameter through P. If there exists a chord having the desired property let $B'B$ be such a chord, the notation chosen so that $B'P = 2BP$. Then we shall have $B'P \cdot BP = A'P \cdot AP$ or $2BP^2 = (AC + CP)(AC - CP) = AC^2 - CP^2$. Now we may construct a segment x such that $2x^2 = AC^2 - CP^2$ as follows: Draw a right triangle with hypotenuse equal to AC and one leg equal to CP; then the length, y, of the other leg is such that $y^2 = AC^2 - CP^2$. Next construct an isosceles right triangle with hypotenuse equal to y; then the legs will be of the required length x, since we have $2x^2 = y^2$. Hence, if the required chord exists its shorter segment is of the constructed length x. If $x < PA$, assuming $PA < PA'$, it therefore follows that no such chord exists. If $x \geqq PA$, the extremity B of the chord may be located by drawing a circle with P as center and x as radius, and noting the points of intersection of this circle with the given circle K; any such intersection is a possible position for B.



The complete ruler and compass construction apparently is made most briefly as follows: Having given the circle K with center at C and the point P, draw the diameter $l_1 = A'A$ through C and P, with P nearer A than A'. With A as center and AC as radius draw a circle K_1 intersecting[1] K in the distinct points D and D'. Draw the line $l_2 = DD'$ intersecting[1] the line l_1 in the point E. With E as center and radius EA draw the circle K_2; with C as center and radius CP draw the circle K_3. Let F be one of the points of intersection[1] of the circles K_2 and K_3. Draw the line $l_3 = AF$. With AF as radius,[2] draw the circle K_4 with center at A, and the circle

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[1] The statement here is put in the usual form of elementary geometry. It is to be noticed that there is a serious defect in the logic here if we fail, as Euclid does under similar circumstances, to prove that the given circles or lines actually do intersect in the required number of points. It can be proved, although the necessary axioms and theorems are not at hand in Euclid, that in the construction we are making the required points of intersection exist in all cases where there is a reference to this foot-note, the proofs being based on axioms and theorems given, for example, by Veblen in Young's *Monographs on Modern Mathematics*.

[2] Here we are simply bisecting the segment AF. There is a logical objection to a common form of procedure, where arcs of unspecified radius are drawn, because it cannot be proved that

K_5 with center at F. Let G be one of the points of intersection[1] of K_4 and K_5. Draw the line $l_4 = GE$ intersecting[1] the line l_3 at the point H. With H as center and radius HA draw the circle K_6; let J be one of its intersections with the line l_4. With AJ as radius and P as center draw the circle K_7 intersecting[3] the circle K in the points B and B_1. Draw the chords $l_5 = BPB'$ and $l_6 = B_1PB_1'$. These are the required chords. Our construction has required the drawing of six lines l_1, l_2, l_3, l_4, l_5, l_6 and seven circles K_1, K_2, K_3, K_4, K_5, K_6, K_7, the locating of ten necessary points A, B, B_1, D', D, E, F, G, H, J. (If one assumes that the center C is not given, as one might from the statement of the problem, the construction is much more difficult.)

Also solved by GEORGE AGINS, E. H. CLARKE, H. N. CARLETON, H. H. DOWNING, EMANUEL GOLDFARB, E. D. GRANT, LAURA GUGGENBUHL, R. A. JOHNSON, MARCIA L. LATHAM, E. W. MARTIN, F. V. MORLEY, A. G. MONTGOMERY, H. L. OLSON, W. B. PIERCE, ARTHUR PELLETIER, JOSEPH ROSENBAUM, ELIJAH SWIFT, CHARLES SCHUMAN, L. G. WELD, C. C. YEN, and the PROPOSER.

2754 [1919, 73]. Proposed by J. W. LASLEY, JR., University of North Carolina.

Given $\bar{x} = \tan^{-1} \dfrac{z}{\sqrt{x^2 + y^2}}$, $\bar{y} = \tan^{-1} \dfrac{y}{x}$, $\bar{z} = \log \sqrt{x^2 + y^2 + z^2}$, solve for x, y, and z in terms of $\bar{x}$, $\bar{y}$, and $\bar{z}$.

SOLUTION BY E. S. SMITH, University of Cincinnati

From the given equations, we have $z/(\sqrt{x^2 + y^2}) = \tan \bar{x}$ (1), $y/x = \tan \bar{y}$ (2), and

$$x^2 + y^2 + z^2 = e^{2\bar{z}}. \tag{3}$$

Substituting the value of y from (2) in (1), gives

$$z = x \sec \bar{y} \tan \bar{x}. \tag{4}$$

Substituting the values of y and z from (2) and (4) in (3), we have

$$x = \pm\, e^{\bar{z}} \cos y \cos \bar{x}. \tag{5}$$

Hence, from (2) and (5),

$$y = \pm\, e^{\bar{z}} \sin \bar{y} \cos \bar{x}. \tag{6}$$

From (4) and (6), we have

$$z = \pm\, e^{\bar{z}} \sin \bar{x}. \tag{7}$$

Hence, the result is $x = \pm\, e^{\bar{z}} \cos \bar{x} \cos \bar{y}$, $y = \pm\, e^{\bar{z}} \sin y \cos \bar{x}$, and $z = \bar{\pm}\, e^{z} \sin \bar{x}$.

Also solved by MARCIA L. LATHAM, E. W. MARTIN, H. L. OLSON, GEORGE PAASWELL, ARTHUR PELLETIER, C. H. RICHARDSON, and D. L. STAMY.

2755 [1919, 73]. Proposed by J. L. RILEY, Stephenville, Texas.

Every number whose square is the sum of the squares of two consecutive integers is equal to the sum of the squares of three integers of which two, at least, are consecutive.

SOLUTION BY ELIJAH SWIFT, University of Vermont.

It is well known that the solution of the equation $a^2 + b^2 = c^2$, where a, b, c are relatively prime integers, is given by the formulas, $a = m^2 - n^2$, $b = 2mn$, $c = m^2 + n^2$, where m and n

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points corresponding to G exist. The radius most easily specified for which the existence of G can be proved is the radius AF. It would not do, for example, to say "let us take any radius greater than a half of AF" because that does not specify an exact radius and it would need to be proved that any radius which was used was actually "greater than a half of AF". The simplest radius to use for the construction and a logical proof is the radius AF.

[3] Here we have assumed as in previous cases that there are points of intersection, but in this case there are none unless $AP \leqq AJ$. Since AJ is one-third of the required chord BB', when it exists, it follows that $AP \leqq \frac{1}{3}BB'$, which is impossible if AP is greater than one-third of a diameter of K. If $AP \leqq \frac{1}{3}$ of a diameter, there always exists a chord which is trisected at P.